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Trends and Recommendation Practices in Didactical Design for Mathematics Learning: A Systematic Literature Review

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ABSTRACT

This systematic literature review examines didactical design in mathematics learning from 2015-2025, analyzing implementation patterns across educational contexts while evaluating key design elements. Following PRISMA guidelines, we analyzed 32 articles from diverse international sources. Results revealed distinct educational level-specific approaches to mathematics instruction, with geometry emerging as the predominant content area. Findings indicate that successful didactical designs align instructional approaches with students' developmental stages: gamification for elementary arithmetic, Realistic Mathematics Education for junior high geometry, problem-based learning for senior high trigonometry, and Socratic questioning for undergraduate calculus. GeoGebra was identified as the most frequently utilized educational tool, highlighting the significance of dynamic mathematics software. This study contributes to mathematics education by providing evidence-based guidance for didactical design implementation while identifying critical research gaps regarding cross-cultural effectiveness, longitudinal impacts, and technology integration. The geographical concentration of studies suggests a need for more diverse international perspectives to strengthen the generalizability of findings.

Keywords: *Didactical Design, Evidence-Based Recommendations, Mathematics Learning, Systematic Literature Review, Trends*

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INTRODUCTION

Mathematics learning requires sophisticated didactic design approaches to develop students' reasoning and problem-solving abilities. Recent research has explored diverse instructional paradigms, including technology integration with GeoGebra (Alves et al., 2020; Sánchez & Munguía, 2020), augmented reality (Fernández-Enríquez & Delgado-Martín, 2020; Schutera et al., 2021; Yaniawati et al., 2023), and gamification (Fauzi et al., 2023; Soboleva et al., 2018). Concurrently, collaborative methodologies and problem-based learning frameworks have enhanced mathematical understanding (Maknun et al., 2020; Sumiaty & Dedy, 2019).

Didactical design as the systematic planning, implementation, and evaluation of instructional interventions that facilitate mathematical understanding (Drijvers et al., 2020; Supriyadi et al., 2023). This process involves integrating content knowledge, pedagogical approaches, and assessment strategies into coherent learning experiences that address students' developmental needs and learning trajectories (Clements & Sarama, 2020; Gravemeijer, 2020).

Despite these advances, significant research gaps persist in understanding the comprehensive landscape of didactical design implementation. Students continue to encounter difficulties in mathematical concept comprehension (Hendriyanto et al., 2024; Kusumaningsih et al., 2020), while teachers face resource limitations and implementation challenges (Aguilar González et al., 2018). Moreover, the field lacks systematic analysis of how effective didactical design varies across different educational levels, content domains, and cultural contexts. This gap is particularly problematic given the developmental differences in students' mathematical thinking across age groups (Clements & Sarama, 2020).

Didactical design operates at the intersection of three theoretical perspectives that frame our analysis. First, drawing on Duval (2006) semiotic representation theory, effective didactical designs facilitate coordination between multiple representational systems a process essential for mathematical understanding. This perspective emphasizes that mathematical objects are accessible only through representations, making the development of representational fluency central to mathematics education. Second, Bakker (2018) content-pedagogy interaction model emphasizes that effective instructional approaches must be calibrated to both content domain and developmental stage. This model rejects one-size-fits-all approaches to mathematics instruction, instead recognizing domain-specific teaching strategies. Third, following Clements & Sarama, (2021) work, we recognize that students follow predictable developmental progressions in mathematical understanding, requiring didactical designs responsive to these trajectories. This perspective highlights the importance of educational level-specific approaches that align with students' cognitive development.

Building on the work of Hiim & Hippe (2001) and Timoshenko et al. (2021), we identify six essential elements of didactical design that form the analytical framework for this study: educational level (the developmental stage and prior knowledge of learners), learning goals (the intended outcomes of the instructional intervention), content focus (the mathematical domain being addressed), pedagogical process (the instructional strategies and approaches employed), instructional media (the tools and resources utilized to support learning), and assessment practices (the methods used to evaluate understanding and progress). These elements interact within a comprehensive didactical design system as illustrated in Figure 1.

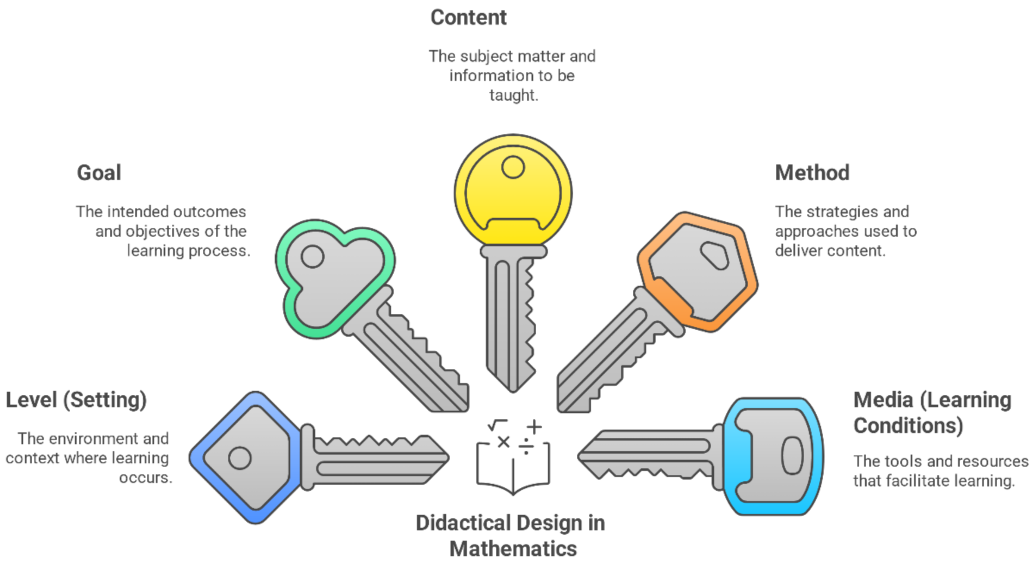

Figure 1: Conceptual Framework showing interconnections between didactical design elements

The interaction between these elements creates what Drijvers et al. (2020) term "didactical configurations" specific arrangements of design elements that address particular learning challenges in mathematics education. By analyzing patterns in these configurations across the literature, we aim to identify evidence-based approaches for different educational contexts.

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This study addresses these gaps by conducting a comprehensive systematic literature review that analyzes research trends across temporal, geographical, and publication dimensions; examines key didactical design elements including educational levels, goals, content, processes, media, and assessment practices; and synthesizes evidence-based recommendations for optimizing didactical design across different educational contexts. Through this approach, we aim to provide mathematics educators with developmentally appropriate, content-specific guidance while identifying critical directions for future research.

This study addresses three interconnected research questions: RQ1) How have publication trends in didactical design research for mathematics learning evolved regarding geographical distribution, journal impact factors, and methodological approaches? RQ2) What patterns emerge in the implementation of didactical design elements (educational level, goals, content, processes, media, and assessment) across different educational contexts? RQ3) What evidence-based recommendations can guide the implementation of didactical designs in mathematics learning across different educational levels?

METHOD

Data Sources and Search Strategy

Three primary databases were used: Google Scholar, ERIC, and Scopus. These databases were selected for their comprehensive coverage of educational research, complementary indexing approaches, and reputation for quality peer-reviewed literature (Pati & Lorusso, 2018). The search strategy was developed in consultation with an academic librarian specializing in educational research. The following Boolean search string was applied to titles, abstracts, and keywords: ("didactic* design" OR "didactic* engineering" OR "teaching experiment" OR "learning design") AND (mathematic* OR geometry OR algebra OR "number sense" OR calculus OR trigonometry) AND (education OR learning OR teaching OR instruction OR classroom). The search was conducted in March 2025 and limited to English-language publications from January 2015 to January 2025. The complete search results yielded 1,216 initial records (920 from Google Scholar, 44 from ERIC, and 252 from Scopus).

Eligibility Criteria

Studies were selected based on pre-defined inclusion and exclusion criteria developed to address the research questions (Table 1).

Table 1: Inclusion and Exclusion Criteria

Criteria	Included	Excluded
Publication Year	Studies published from January 2015 to January 2025	Studies published before 2015
Document Type	Articles and Conference papers	Reviews, book chapters, editorials, etc.
Source Type	Scopus-indexed international journals or proceedings	Non-indexed journals, book series, trade publications
Language	English	Other languages
Focus and Context	Studies focused on didactic design in mathematics learning	Studies not related to didactic design or mathematics education
Accessibility	Full-text accessible	Full-text inaccessible
Research Design	Original research with didactic design components and empirical evidence	Theoretical papers without empirical evidence
Educational Level	Formal education settings	Non-formal education contexts
Geographical Scope	All countries and regions	None

Study Selection Process

The selection process followed the PRISMA flow diagram (Figure 2). After removing duplicates, two independent reviewers screened titles and abstracts against the inclusion criteria. Full-text articles were then evaluated against complete eligibility criteria by the same reviewers. The systematic screening process reduced the initial 1,216 studies to 32 articles that met all inclusion criteria (see **Appendix** for detail).

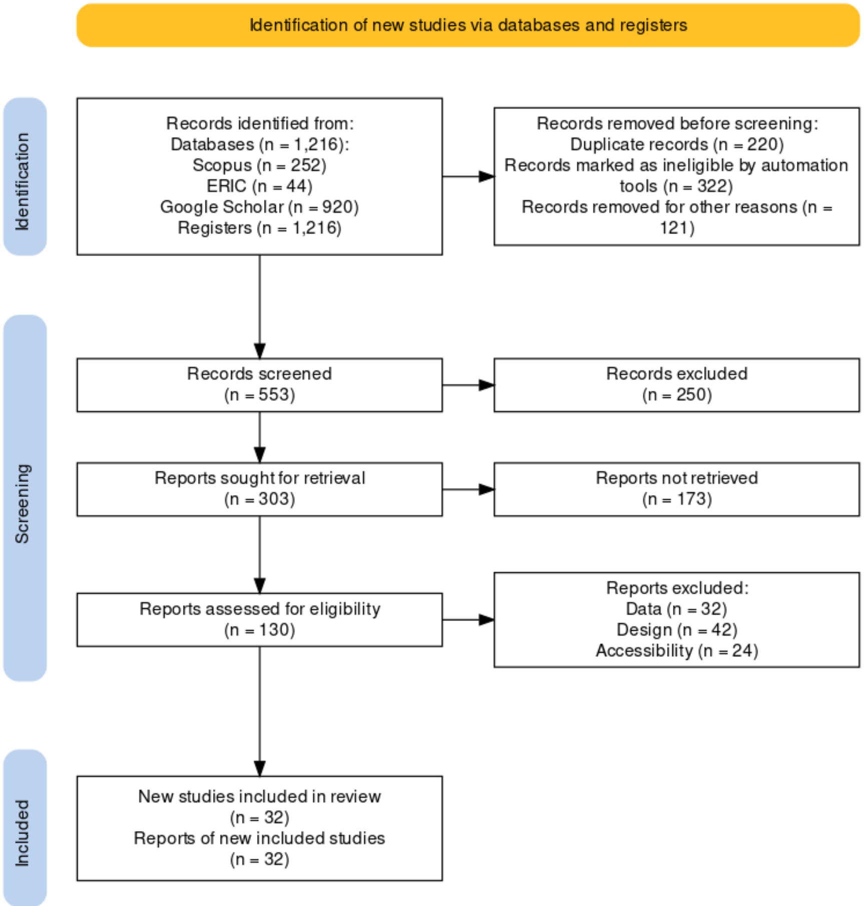

Figure 2: PRISMA flow diagram showing the study selection process

FINDING AND DISCUSSION

RESEARCH RESULT

RQ1: Trends in Publication Years, Countries, and Journal Impact

Analysis of publication patterns revealed an increasing trend in didactical design research, with peak productivity in 2021 (22%), followed by 2019-2020 (19% each). This pattern aligns with what Drijvers et al. (2020) describe as "punctuated equilibrium" in educational research, where periods of relative stability are interrupted by surges of innovation. The 2021 peak coincides with post-COVID educational adaptation, representing what Bakker (2018) terms a "design response phase" following disruption.

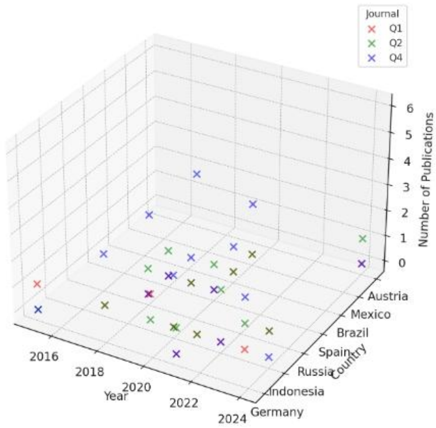

Figure 3: Distribution by Year, Country, and Journal Quality

Geographical Geographical analysis revealed significant variation in research activity, with Indonesia contributing 69%, followed by Spain (10%), Germany (6%), and other countries (15%). This reflects Cai & Hwang's (2023) concept of “contextual embeddedness,” where national policies and cultural factors shape research priorities. Indonesia’s dominance is due to the “Merdeka Belajar” philosophy promoting innovation (Abidah et al., 2020), competency-based curriculum reforms, and stronger support for research publication. However, this concentration limits cross-cultural generalizability, highlighting the need for more diverse international perspectives. Publication quality analysis showed that most studies were published in Q4 journals (66%), with fewer in Q2 (25%) and Q1 (9%), reflecting structural inequalities in academic knowledge dissemination, such as language barriers and publication bias toward Western contexts (Arsyad et al., 2019).

RQ2: Patterns in Didactical Design Elements

Educational Levels and Implementation Goals

Junior high school emerged as the primary implementation context (31%), followed by elementary school (25%), senior high school (19%), and undergraduate level (16%), with teacher education comprising the remaining 9% based on figure 4. This distribution aligns with Barrouillet (2015) developmental framework identifying early adolescence as a critical transition period from concrete to formal operations in mathematical thinking. The emphasis on junior high school appears driven by several factors: this stage represents a critical shift from concrete to abstract mathematical thinking, requiring carefully designed learning experiences (Drijvers et al., 2020); junior high mathematics bridges elementary concepts and advanced applications, developing foundations for higher-level mathematics (Kieran, 2019); and this stage often sees the emergence of mathematics anxiety and disengagement, making effective didactical design particularly important (Gueudet, 2019a). Through the lens of Duval (2006) semiotic theory, this transition period requires particular support as students navigate between increasingly abstract representational systems. The concentration of research at this level suggests recognition of what Clements & Sarama (2020) call a “critical window” for mathematical development.

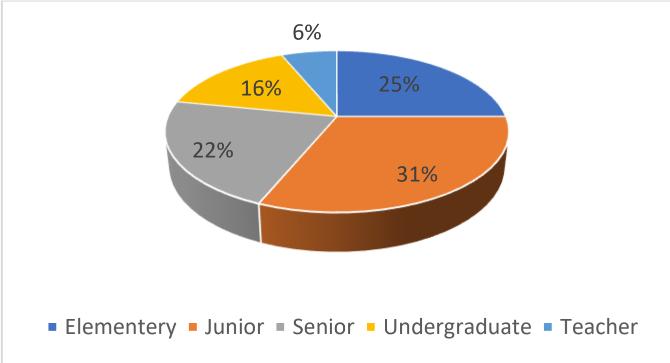

Figure 4: Educational Level

Analysis of implementation goals revealed that addressing learning obstacles (25%) and enhancing conceptual understanding (25%) were the predominant objectives based on figure 5. This distribution reflects the field's recognition of what Duval (2006) terms "semiotic complexity" in mathematics learning the challenges students face in coordinating between different representational systems. The focus on learning obstacles indicates recognition that mathematical difficulties often stem from specific conceptual barriers rather than general cognitive limitations. These obstacles frequently occur at what Goldin (2018) calls "representational disconnections" where students struggle to coordinate between different mathematical representations. Didactical designs addressing these obstacles employed what Bakker (2018) identifies as "content-dependent pedagogical approaches," recognizing that different mathematical domains require domain-specific teaching strategies.

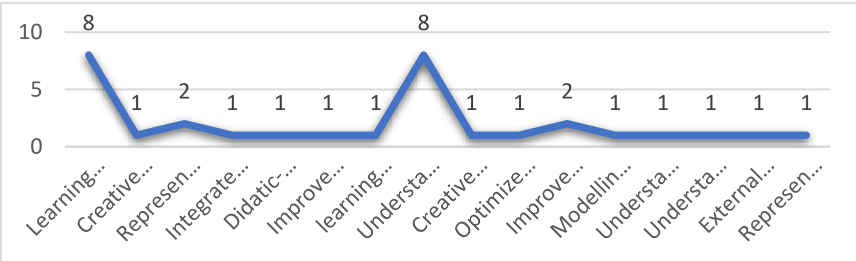

Figure 5: Implementation Goals

Content Focus and Pedagogical Processes

Geometry dominated as the primary content area (47%, 15 studies), significantly outpacing algebra (22%, 7 studies), numbers (19%, 6 studies), and others (12%, 4 studies) based on figure 6. This strong emphasis on geometry warrants closer examination. Geometry's prevalence appears driven by several factors: it provides visual, tangible approaches to mathematics that engage learners who may struggle with purely abstract reasoning (Jones & Tzekaki, 2016); dynamic geometry software creates particularly powerful learning opportunities through interactive visualization and exploration (Wegner, 2022); geometric reasoning bridges concrete and abstract thinking, making it particularly valuable during the junior high transition period; and geometric concepts offer rich connections to physical experiences and authentic applications (Sinclair et al., 2016). This content distribution suggests that geometry serves as what Bakker (2018) terms a “pedagogical bridge” a content domain that effectively connects concrete and abstract mathematical thinking. The visual nature of geometry aligns with Duval (2006) emphasis on coordinating between representational systems, as geometric learning often involves connections between visual, symbolic, and verbal representations.

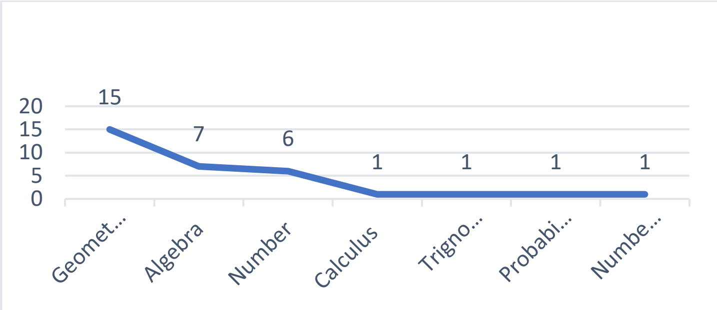

Figure 6: Content Focus

Dominant approach is gamification (13%, 4 studies) based on figure 7. This diversity reflects the field’s alignment with contemporary learning theories emphasizing student engagement and authentic practices (Schoenfeld, 2020). The prominence of gamification, especially at the elementary level, highlights the importance of motivational factors in mathematics learning, aligning with Goldin’s (2018) emphasis on the affective role in fostering cognitive receptiveness.

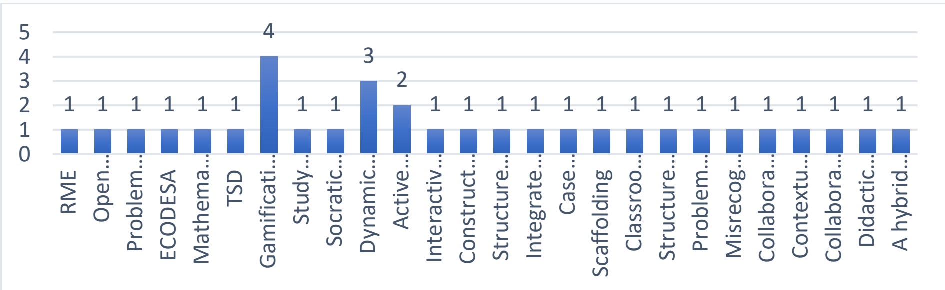

Figure 7: Pedagogical Process

Instructional Media and Assessment Practices

GeoGebra emerged as the most frequently utilized educational tool (15.6%, 5 studies), based on figure 8. The prominence of GeoGebra highlights the transformative potential of dynamic mathematics software in supporting representational fluency. GeoGebra’s prevalence aligns with Duval (2006) emphasis on conversions between semiotic registers, as the software specifically enables students to visualize dynamic connections between algebraic and geometric representations. This integrates with Bakker (2018) content-pedagogy interaction model by demonstrating how specific technological tools can mediate particular mathematical content domains more effectively than others.

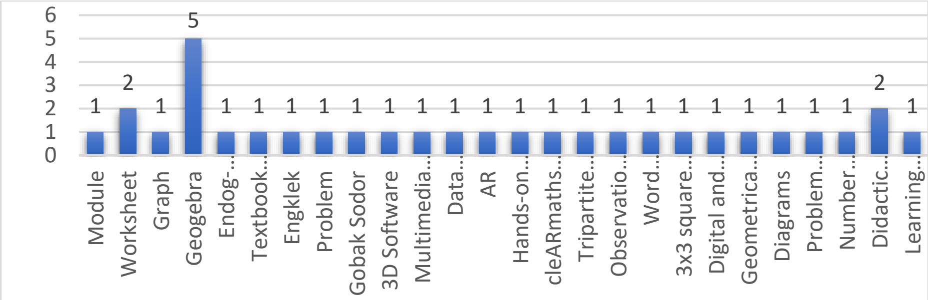

Figure 8: Instructional Media

The versatility of GeoGebra appears particularly valuable for several reasons: the software integrates algebraic, geometric, and numerical representations in a unified interface (Alves et al., 2020); students can manipulate mathematical objects in real-time, testing conjectures and observing patterns (Gökçe & Güner, 2022); the tool supports student-centered exploration and discovery (Kaya & Öcal, 2018); and GeoGebra facilitates connections between abstract concepts and real-world phenomena (Pfeiffer, 2017). These features exemplify what Faggiano et al. (2017) describe as "representational technologies" that support the cognitive processes central to mathematical understanding.

Assessment practices showed a preference for combined approaches, with 56% (18 studies) using both test and non-test methods, while 25% (8 studies) relied exclusively on tests and 19% (6 studies) on non-test approaches based on figure 9. This integration reflects what Black & Wiliam, (2018) characterize as "assessment for learning" rather than merely "assessment of learning." The prevalence of combined assessment approaches indicates recognition that mathematical competence encompasses both what Goldin (2018) terms "external representational competence" (measurable through tests) and "internal representational systems" (observable through performance tasks, discussions, and other non-test methods). This assessment pattern supports Verschaffel et al. (2020) assertion that mathematical understanding requires evaluation of both procedural fluency and conceptual understanding across multiple representational contexts.

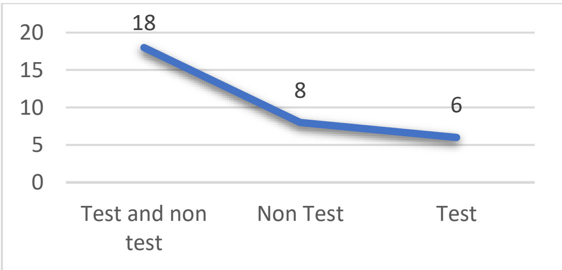

Figure 9: Assessment Practices

RQ3: Evidence-Based Recommendations for Didactical Design

Analysis of successful implementations across studies yielded evidence-based recommendations for optimizing didactical design across educational levels based on figure 10. These recommendations represent what Gravemeijer et al. (2017) term "design principles for developmental progression" evidence-based guidance for creating learning environments that support students' mathematical development across the educational trajectory.

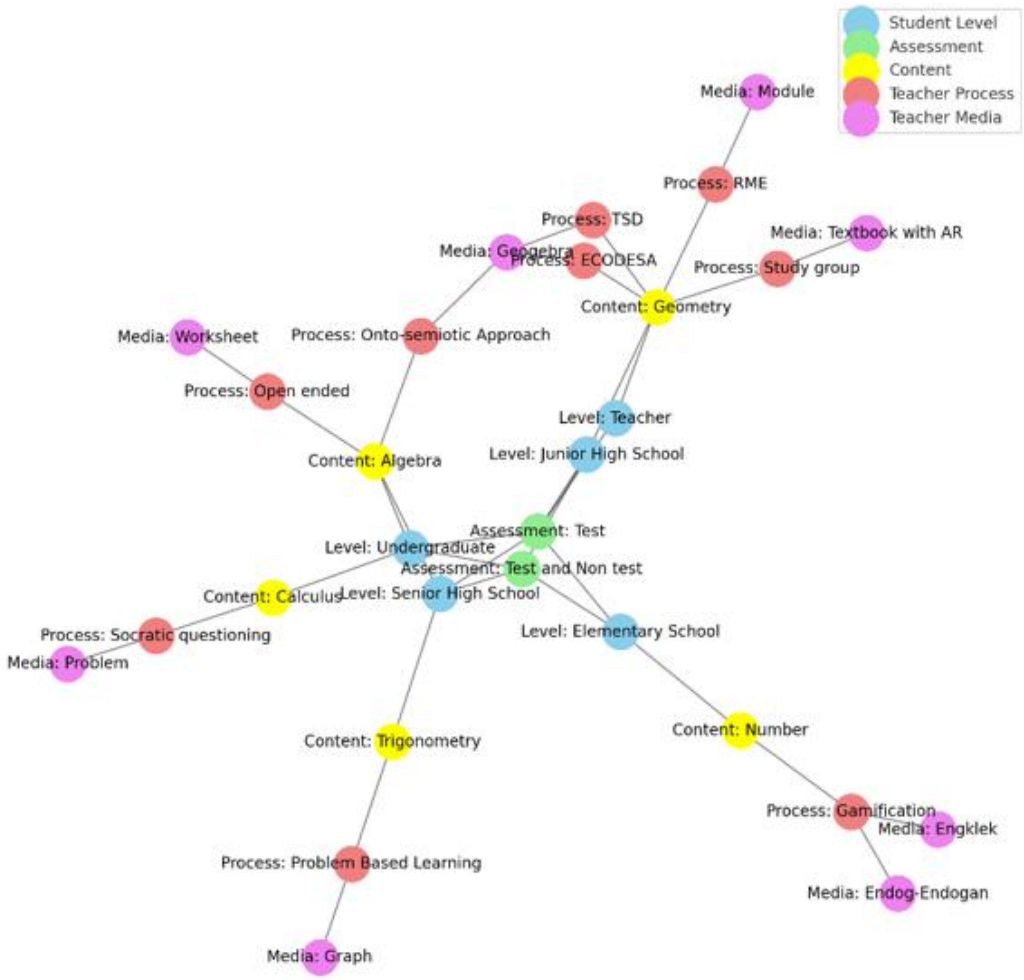

Figure 10: Mapping of Combinations of Didactical Design Elements

The pattern of recommendations across educational levels reflects what Clements & Sarama (2020) describe as a "developmental progression of representational competence" in mathematics education. Each recommended approach addresses the particular representational challenges identified by Duval (2006) as central to mathematical understanding at that level from establishing foundational representational systems at the elementary level to coordinating multiple sophisticated representations at the undergraduate level.

Table 2: Evidence-Based Recommendations by Educational Level

Educational Level	Recommended Content	Suggested Processes	Rationale
Elementary School	Numbers and Arithmetic	Gamification	Aligns with concrete thinking and motivation needs
Junior High School	Geometry	RME with Study Groups	Bridges concrete and abstract thinking; leverages social development
Senior High School	Trigonometry	Problem-Based Learning	Develops abstract reasoning and application to authentic contexts
Undergraduate	Calculus	Socratic Questioning	Promotes deep conceptual understanding and metacognition

The combinations of subject characteristics (level), content, and interventions provided by educators show a mapping as presented in Figure 9. Based on this mapping, several recommended didactic situations are outlined as shown in Table 2. The analysis results indicate that at the elementary level, didactic design is suitably used for number content with fun learning interventions involving games (Fauzi et al., 2023; Supriadi, 2022). At the junior high level, didactic design is suitable for geometry content with RME (Kusumaningsih et al., 2020) with study group (Yaniawati et al., 2023). At the senior high school level, didactic design is effectively used for trigonometry content with problem-based learning (Rosjanuardi & Jupri, 2020). At the higher education level, didactic design is appropriate for calculus with sacrotic questoning interventions (Puspita et al., 2023). For teacher training, it is recommended to employ the theory of situated didactic (TSD) approach on geometry materials (Alves et al., 2020). Nevertheless, the selection of combinations and levels, content, and processes is also possible. However, this study focuses on the differences in levels, content, and processes, ensuring that there is no overlap between the elements.

DISCUSSION

RQ1. The Trends in Publication Years, Countries, and Journal Qualities of Didactic Design Research in Mathematics learning

Several factors contribute to the significant amount of Didactical Design Research (DDR) originating from Indonesia, particularly peaking in 2021 post-COVID-19. The diverse cultural landscape and unique educational system necessitate research tailored to the Indonesian context, considering multicultural education values and the impact of the pandemic (Tarigan et al., 2022). Indonesia's emphasis on mathematics education and the development of students' mathematical thinking skills drive the prevalence of DDR, with studies highlighting effective instructional strategies (Supriyadi et al., 2023). The COVID-19 pandemic and the "Merdeka Belajar" curriculum philosophy prompted researchers to explore innovative didactical approaches to address educational challenges in the new normal era, optimizing online learning and improving outcomes in subjects like mathematics (Marfuah et al., 2022).

Most DDR in Indonesia is published in low-impact journals or proceedings due to several factors. Indonesian researchers prioritize practical relevance and immediate classroom application, often choosing journals with strong practitioner readership but lower impact factors (Achsan et al., 2019). Language barriers and accessibility issues lead researchers to prefer local or regional journals over high-impact international ones (Arsyad et al., 2019). Limited funding and resources also restrict access to high-impact journals, making low-cost or open-access options more appealing (Fry et al., 2023). Additionally, research focused on local educational challenges may not attract high-impact journals' interest, and the developing research culture in Indonesia means many researchers are still building their capabilities (Hanami et al., 2023).

RQ2. Elements of Didactical Design Trends Based on Educational Levels, Goals, Content, Processes, Media, and Assessments for Mathematics Learning *Level*

Primary reasons for the emphasis on didactical design in junior high school is the developmental stage of the students. At this age, students are transitioning from concrete to abstract thinking, and they require carefully designed learning experiences to support this cognitive shift (Barrouillet, 2015). Didactical design allows educators to create tasks and activities that are developmentally appropriate and aligned with students' cognitive abilities (Drijvers et al., 2020). By considering the level of abstraction and complexity, teachers can scaffold learning and facilitate the development of mathematical reasoning and problem-solving skills (Radford, 2021).

Another factor that contributes to the implementation of didactical design in junior high school is the need to bridge the gap between elementary and high school mathematics. Junior high school is a critical period for laying the foundation for advanced mathematical concepts and skills (Kieran, 2019). Didactical design enables educators to create a coherent and well-structured curriculum that builds upon prior knowledge and prepares students for the challenges of high school mathematics (Cai et al., 2020). By carefully selecting and sequencing content, teachers can ensure that students develop a strong conceptual understanding and procedural fluency (Marton & Pang, 2008)

Moreover, junior high school is a time when students' attitudes towards mathematics can significantly influence their future academic and career choices. Negative experiences and perceptions of mathematics at this stage can lead to disengagement and avoidance of the subject in later years (Olsson & Granberg, 2022). Didactical design allows educators to create engaging and meaningful learning experiences that foster positive attitudes towards mathematics (Gueudet, 2019b; Pepin et al., 2017). By incorporating real-world applications, collaborative activities, and technology-enhanced learning, teachers can motivate students and demonstrate the relevance of mathematics in their lives (Faggiano et al., 2017; Pepin et al., 2017).

The implementation of didactical design in junior high school also aligns with the increasing emphasis on student-centered learning and inquiry-based approaches in mathematics learning. Didactical design enables teachers to create tasks and activities that promote active engagement, exploration, and discovery (Schoenfeld, 2020). By providing opportunities for students to work collaboratively, engage in mathematical discourse, and construct their own understanding, didactical design supports the development of critical thinking, communication, and problem-solving skills.

Goals

Primary reasons for using didactical design to address learning obstacles is the recognition that students often struggle with certain mathematical concepts and procedures. Learning obstacles can arise from various sources, such as misconceptions, inadequate prior knowledge, or cognitive overload (Radford, 2021). By identifying and analyzing these obstacles, educators can design targeted interventions that help students overcome their difficulties and develop a deeper understanding of the subject matter (Cevikbas et al., 2024). Didactical design allows for the creation of learning experiences that are tailored to the specific needs of students, taking into account their cognitive development, prior knowledge, and potential misconceptions.

Another reason for the emphasis on didactical design in addressing learning obstacles is the growing recognition of the importance of conceptual understanding in mathematics learning. Research has shown that students who develop a deep conceptual understanding of mathematical concepts are better equipped to apply their knowledge to new situations, solve complex problems, and engage in higher-order thinking (Drijvers et al., 2020). Didactical design enables educators to create learning experiences that prioritize conceptual understanding over procedural fluency, encouraging students to explore mathematical ideas, make connections, and construct their own meaning (Schoenfeld, 2020). By carefully designing tasks and activities that promote conceptual understanding, educators can help students develop a more robust and flexible knowledge base.

Moreover, didactical design allows for the integration of multiple representations and the use of real-world contexts, which can be particularly effective in addressing learning obstacles and improving understanding. By presenting mathematical concepts in various forms, such as visual, verbal, and symbolic representations, educators can cater to different learning styles and help students develop a more comprehensive understanding of the subject matter (Kieran, 2019). The use of real-world contexts and authentic problems can make mathematics more meaningful and relevant to students, increasing their motivation and engagement (Gueudet, 2019b). Didactical designs that incorporate multiple representations and real-world applications can help students connect abstract mathematical concepts to concrete experiences, facilitating the development of deeper understanding.

Content

One of the primary reasons for the emphasis on geometry in didactical design is its fundamental role in mathematics learning. Geometry is a core component of mathematics curricula worldwide, and it provides a rich context for developing spatial reasoning, visualization skills, and problem-solving abilities (Sinclair et al., 2016). Geometric thinking is essential for understanding and navigating the physical world, and it has applications in various fields, such as art, architecture, engineering, and computer graphics (Jones & Tzekaki, 2016). By designing learning experiences that focus on geometric concepts and reasoning, educators can help students develop a strong foundation for future mathematical learning and real-world problem-solving.

Another reason for the prevalence of geometry content in didactical design is its potential for fostering student engagement and motivation. Geometry offers a visual and tangible approach to mathematical concepts, which can be particularly appealing to learners who struggle with abstract reasoning (Russell & Potter, 2022). By incorporating manipulatives, dynamic geometry software, and interactive tasks, didactical designs in geometry can create opportunities for exploration, discovery, and hands-on learning (Trgalová, 2022). These engaging learning experiences can help students develop a positive attitude towards mathematics and increase their confidence in their mathematical abilities (Biza et al., 2020).

Moreover, geometry provides a rich context for developing mathematical thinking and reasoning skills. Didactical designs in geometry often involve tasks that require students to analyze properties of shapes, construct arguments, and justify their reasoning (Sinclair et al., 2017). These activities promote the development of critical thinking, logical reasoning, and communication skills, which are essential for success in mathematics and other disciplines (Schoenfeld, 2020). By carefully designing tasks that challenge students to think deeply about geometric concepts and relationships, educators can support the development of higher-order thinking skills and mathematical proficiency (Cai & Hwang, 2023).

The use of geometry content in didactical design also aligns with the increasing emphasis on technology integration in mathematics learning. Dynamic geometry software, such as GeoGebra and Cabri, has become increasingly popular in recent years, offering new opportunities for exploring geometric concepts and relationships (Fernández-Enríquez & Delgado-Martín, 2020). These tools allow students to manipulate and investigate geometric objects, formulate conjectures, and test hypotheses (Wegner, 2022). Didactical designs that incorporate technology-enhanced learning environments can promote student engagement, collaboration, and self-directed learning (Gueudet & Pepin, 2023).

Process Primary reasons for the growing interest in gamification within didactical design is its potential to increase student engagement and motivation. Gamification involves the application of game design elements, such as points, badges, leaderboards, and challenges, to non-game contexts (Dichev & Dicheva, 2017). By incorporating these elements into mathematics learning experiences, educators can create a more engaging and enjoyable environment that encourages active participation and persistence (Jagušt et al., 2018). Studies have shown that well-designed gamification strategies can improve student attitudes towards mathematics, increase their willingness to take on challenges, and foster a growth mindset (Treiblmaier et al., 2018; Yildirim, 2017).

Dynamic learning environments, such as interactive simulations, virtual manipulatives, and digital platforms, have also gained prominence in didactical design due to their ability to enhance conceptual understanding and problem-solving skills. These environments allow students to explore mathematical concepts, manipulate objects, and observe the consequences of their actions in real-time (Zengin, 2019). By providing multiple representations and opportunities for experimentation, dynamic learning environments can help students develop deeper insights into mathematical relationships and structures (Cevikbas & Kaiser, 2021). Studies have shown that well-designed dynamic learning environments can promote the development of higher-order thinking skills, such as reasoning, justification, and generalization (Van Voorhis & Paris, 2019).

The use of gamification and dynamic learning environments in didactical design also aligns with the growing emphasis on personalized and adaptive learning in mathematics learning. These approaches enable educators to tailor learning experiences to individual student needs, preferences, and learning styles (Plass et al., 2020). By leveraging data analytics and intelligent tutoring systems, gamification and dynamic learning environments can provide personalized feedback, recommendations, and learning paths that optimize student progress and achievement (Vahdat et al., 2015). This personalization can help address the diverse needs of learners and promote equity in mathematics learning (Reich et al., 2017).

Media

Using GeoGebra in didactical design is its versatility and dynamic nature. GeoGebra is a free, open-source software that combines geometry, algebra, calculus, and statistics in a single, user-friendly interface (Zöchbauer et al., 2021). This integration of multiple mathematical representations allows students to explore and manipulate mathematical objects in real-time, fostering a deeper understanding of underlying concepts and relationships (Gökçe & Güner, 2022b). Studies have shown that the dynamic and interactive features of GeoGebra can help students visualize abstract concepts, make conjectures, and test hypotheses, promoting active learning and mathematical reasoning (Ekwue et al., 2023; Mensah et al., 2023).

Moreover, GeoGebra aligns well with the principles of constructivism and inquiry-based learning, which emphasize the importance of student-centered exploration and discovery (Yenmez et al., 2017). Didactical designs that incorporate GeoGebra can provide students with opportunities to construct their own knowledge through hands-on experimentation and problem-solving (Korenova et al., 2019). By allowing students to manipulate variables, observe the consequences of their actions, and receive immediate feedback, GeoGebra can foster a sense of ownership and autonomy in the learning process (Khali & Khalil, 2019). This student-centered approach can increase motivation, engagement, and persistence in mathematics learning (Kaya & Öcal, 2018).

Another reason for the prevalent use of GeoGebra in didactical design is its capacity to support multiple representations and mathematical modeling. GeoGebra allows students to seamlessly switch between algebraic, graphical, and tabular representations of mathematical objects, promoting a more holistic understanding of concepts (Pfeiffer, 2017). This ability to connect different representations can help students develop fluency in translating between various forms of mathematical expression, a crucial skill for problem-solving and application (Karakus et al., 2022). Additionally, GeoGebra's modeling capabilities enable students to create and explore realistic scenarios, bridging the gap between abstract mathematical concepts and real-world phenomena (Çekmez, 2020). By engaging in mathematical modeling with GeoGebra, students can develop critical thinking, creativity, and problem-solving skills that are essential for success in the 21st century (Geiger et al., 2018). Overal, media really contributes to making it easier for teachers to deliver material in class so that learning the material becomes more effective and efficient (fahruh, 2024; Juhaevah et al., 2025; Juhaevah, 2021).

Assessment

Reasons for the use of both test and non-test assessments in didactical design is the recognition that different assessment methods provide complementary insights into student learning. Traditional test-based assessments, such as written exams and quizzes, are often used to measure students' content knowledge, procedural skills, and ability to apply mathematical concepts to solve problems (Leder & Forgasz, 2018). These assessments provide a standardized and objective measure of student performance, allowing for comparisons across individuals and groups (Sussman & Wilson, 2019). However, test-based assessments may not fully capture the depth and complexity of student understanding, particularly in areas such as mathematical reasoning, problem-solving strategies, and communication skills (Lesh et al., 2020)

Non-test assessments, such as observations, interviews, portfolios, and performance tasks, offer a more holistic and authentic approach to evaluating student learning (Zhang et al., 2021). These assessments allow educators to gather evidence of students' mathematical thinking processes, conceptual understanding, and ability to apply knowledge in real-world contexts (Suurtamm et al., 2016). Non-test assessments can provide valuable insights into students' attitudes, beliefs, and dispositions towards mathematics, which are critical factors in their long-term engagement and success in the subject (Hannula et al., 2016). By using a combination of test and non-test assessments, educators can obtain a more comprehensive and nuanced understanding of student learning, enabling them to provide targeted support and interventions (Scoular et al., 2020).

Moreover, the use of both test and non-test assessments aligns with the principles of formative assessment, which emphasizes the continuous monitoring and adjustment of teaching and learning based on evidence of student understanding (Black & Wiliam, 2018). Formative assessment involves the use of a variety of assessment methods to elicit student thinking, provide feedback, and guide instructional decision-making (James, 2017). Test-based assessments can be used formatively by providing timely feedback on student performance, identifying areas of strength and weakness, and informing the design of subsequent learning. Non-test assessments, such as classroom discussions, student self-reflections, and peer feedback, can provide ongoing evidence of student learning and enable teachers to adapt their instruction in real-time activities (Lamberg et al., 2020). By using both test and non-test assessments formatively, educators can create a more responsive and personalized learning environment that supports student growth and achievement (Cai & Mamlok-Naaman, 2020).

RQ3. The Recommendation Practice Based on Didactical Design Elements for Mathematics Learning in The Future

At the elementary school level, the focus is on developing a strong foundation in numbers and arithmetic skills (Fauzi et al., 2023; Silo et al., 2021; Supriadi, 2022). Gamification is recommended as an effective approach to engage young learners and promote a positive attitude towards mathematics (Byun & Joung, 2018; Jagušt et al., 2018). By incorporating game-like elements, such as rewards, challenges, and interactive activities, educators can create a motivating and enjoyable learning environment that encourages active participation and promotes understanding of number concepts (Dichev & Dicheva, 2017; Sailer & Homner, 2020).

For junior high school students, the emphasis shifts to geometry (Fernández-Enríquez & Delgado-Martín, 2020; Kusumaningsih et al., 2020; Rohaeti et al., 2019). The suggested process is Realistic Mathematics learning (RME) with study groups (Kusumaningsih et al., 2020; Laurens et al., 2017; Sumirattana et al., 2017). RME emphasizes the use of real-world contexts and student-centered learning, allowing students to construct their own understanding of geometric concepts through exploration and problem-solving (Jones & Tzekaki, 2016). Study groups provide opportunities for collaborative learning, where students can share ideas, challenge each other's thinking, and develop a deeper understanding of geometric principles (Boaler et al., 2018)

At the senior high school level, trigonometry is the recommended content. Problem-based learning (PBL) is suggested as an effective instructional approach (Marchy et al., 2022; Ramadhany et al., 2025; Sern et al., 2015). PBL presents students with complex, real-world problems that require the application of trigonometric concepts and problem-solving skills (Wilder, 2015). By engaging in authentic problem-solving experiences, students develop critical thinking, collaboration, and communication skills while deepening their understanding of trigonometric (Nfon, 2013).

For undergraduate students, calculus is the recommended content (Puspita et al., 2023). Socratic questioning is suggested as a powerful technique to facilitate deep understanding and critical thinking (Neenan, 2009 Vittorio et al., 2022). Socratic questioning involves engaging students in a dialogic process of inquiry, where the instructor poses thought-provoking questions that challenge students' assumptions, stimulate reflection, and promote a deeper exploration of calculus concepts (Puspita et al., 2023). This approach encourages active learning, fosters critical thinking skills, and helps students develop a robust conceptual understanding of calculus (Katsara & De Witte, 2019).

For teacher training, the implementation of didactic design is intended to analyze students' learning situations and enhance teachers' pedagogical abilities in geometry materials (Alves et al., 2020) particularly in terms of selecting appropriate interventions for the learning process that will be subsequently implemented in the classroom (Anthony & Walshaw, 2023). Teachers play a crucial role in assisting students to achieve their learning objectives. Therefore, teachers must continually enhance their competencies, one way being through attending relevant competency development training programs (Ramos-Rodríguez et al., 2021).

The findings of the systematic literature review, emphasizing the importance of aligning content and instructional strategies with the developmental stages and learning needs of students. The recommended approaches, such as gamification, RME with study groups, PBL, and Socratic questioning, have the potential to enhance mathematics learning and promote deep understanding, critical thinking, and problem-solving skills. However, educators must carefully consider the specific context of their classrooms and adapt these strategies accordingly. The integration of technology can further enhance the effectiveness of these approaches and provide additional opportunities for engagement and personalized learning such as GeoGebra. Didactical design is most effective when the assessment includes both test and non-test methods.

Theoretical Implications and Future Research Directions

Theoretical Contributions

This review contributes to the theoretical understanding of didactical design in mathematics education. Our findings support Duval's (2006) semiotic representation theory, showing how successful designs coordinate multiple representational systems. The relationship between educational levels, content domains, and pedagogical approaches aligns with Bakker's (2018) content-pedagogy interaction model, emphasizing the need for approaches tailored to both content and developmental stage. The progressive nature of effective designs aligns with Clements & Sarama's (2020) concept of developmental learning trajectories, highlighting the evolution of instructional strategies. We propose an integrated model that aligns representational systems, content, and developmental stages through appropriate pedagogical approaches and technological mediation.

Limitations

Several limitations should be considered. The dominance of Indonesian studies (69%) limits cross-cultural generalizability. The prevalence of Q4 journals (66%) raises concerns about methodological rigor, though the research designs were generally sound. Limiting the review to English-language publications may have excluded valuable non-English research.

Future Research Directions

We recommend cross-cultural comparative studies to examine didactical principles across diverse educational systems, addressing universal structures and culturally-specific learning processes. Longitudinal research is needed to explore the sustained impact of didactical innovations (Clements & Sarama, 2020). Future studies should explore applications beyond dynamic geometry software, particularly augmented reality and AI for personalized learning experiences. Additionally, implementation science should investigate factors that facilitate or hinder the success of evidence-based designs in diverse educational contexts, addressing the research-practice gap in mathematics education.

CONCLUSION

This systematic review analyzed trends in didactical design for mathematics learning from 2015-2025, synthesizing 32 studies. Key findings highlight effective strategies tailored to educational levels: gamification for elementary number concepts, RME with study groups for junior high geometry, problem-based learning for senior high trigonometry, and Socratic questioning for undergraduate calculus. These recommendations reflect developmental and content-specific needs, offering actionable guidance for educators. Geometry emerged as the dominant subject, supporting Duval's theory on the coordination of representational systems in mathematical understanding. The widespread use of GeoGebra and combined assessment approaches underscores the importance of dynamic tools and balanced skill development.

Content analysis revealed geometry as the dominant subject area, reflecting its unique position at the intersection of visual, spatial, and abstract reasoning. This content preference, alongside the prevalence of multiple representation approaches, supports Duvals' theory that mathematical understanding develops through coordination between representational systems. The prominence of GeoGebra as an instructional tool highlights the value of dynamic mathematics software in facilitating representational fluency. The preference for combined assessment approaches demonstrates recognition that mathematical competence encompasses both procedural fluency and conceptual understanding.

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APPENDIX

No.	Research (Year)	Level	Goal	Content	Process	Media	Assessment	Country	Journal
1	(Kusumaningsih et al., 2020)	Junior High School	Learning obstacle	Geometry	RME	Module	Test and Non test	Indonesia	Q4
2	(Adharini & Herman, 2021)	Senior High School	Creative thinking and self confidence	Algebra	Open ended dan Contextual Approach	Worksheet	Test and Non test	Indonesia	Q4
3	(Rosjanuardi & Jupri, 2020)	Senior High School	Representation	Trigonometry	Problem Based Learning	Graph	Test	Indonesia	Q4
4	(Sánchez & Munguía, 2020)	Teacher	Integrated technology	Geometry	ECODESA	Geogebra	Test	Mexico	Q4
5	(Herrera-García & Dávila-Araiza, 2020)	Undergraduat e Student	Didatic-Mathematical Knowledge and Competence	Algebra	Mathematical Knowledge and Competence model	Geogebra	Test and Non test	Mexico	Q4
6	(Alves et al., 2020)	Teacher	Improve didactic competence	Geometry	TSD	Geogebra	Test	Brazil	Q4
7	(Supriadi, 2022)	Elementary School	learning mathematics easy and flexible	Number	Gamification	Endog-Endogan	Test and Non test	Indonesia	Q4
8	(Yaniawati et al., 2023)	Junior High School	Understanding	Geometry	Study group	Textbook with AR	Test and Non test	Indonesia	Q2
9	(Fauzi et al., 2023)	Elementary School	Understanding	Number	Gamification	Engklek	Test	Indonesia	Q4
10	(Puspita et al., 2023)	Undergraduat e Student	Understanding	Calculus	Socratic questioning	Problem	Test	Indonesia	Q4
11	(Lindenbauer et al., 2024)	Elementary School	Understanding	Geometry	Dynamic learning environment	Geogebra	Non test	Austria	Q2
12	(Nur'Aeni et al., 2019)	Elementary School	Learning obstacle	Geometry	Gamification	Gobak Sodor	Test	Indonesia	Q4
13	(Setiadi et al., 2017)	Senior High School	Understanding	Geometry	Active learning	3D Software	Test and Non test	Indonesia	Q4

No.	Research (Year)	Level	Goal	Content	Process	Media	Assessment	Country	Journal
14	(Rohaeti et al., 2019)	Junior High School	Creative thinking	Geometry	Interactive teaching	Multimedia PPT with VB	Test and Non test	Indonesia	Q4
15	(Godino et al., 2019)	Teacher	Optimize learning	Probability	Constructivist and objectivist approach	Data analysis project	Test and Non test	Spain	Q4
16	(Fernández-Enríquez & Delgado-Martín, 2020)	Junior High School	Improve teaching and learning process	Geometry	Dynamic learning environment	AR	Test and Non test	Spain	Q2
17	(Silo et al., 2021)	Elementary School	Learning obstacle	Number	Structured learning environment	Hands-on activities, audio and visual aids, and traditional written materials	Test and Non test	Indonesia	Q4
18	(Schutera et al., 2021)	Senior High School	Improve teaching and learning process	Geometry	Integrate learning	cleARmaths application	Non test	Germany	Q2
19	(Soboleva et al., 2018)	Elementary School	Modelling activities	Algebra	Gamification	Tripartite interactions	Test and Non test	Russia	Q2
20	(Aguilar González et al., 2018)	Teacher	Understand the professional knowledge	Geometry	Case study with reflective	Observation sheet	Non test	Spain	Q2
21	(Prediger & Krägeloh, 2015)	Junior High School	Understanding and problem solving	Algebra	Scaffolding	Word problem cracker	Non test	Germany	Q1
22	(Prabawanto & Mulyana, 2017)	Undergraduate Student	Understanding	Geometry	Dynamic learning environment	3x3 square grid and numerical sets	Non test	Indonesia	Q4
23	(Jatisunda et al., 2021)	Undergraduate Student	Understanding	Geometry	Classroom practice with rigorous	Digital and print media	Non test	Indonesia	Q4
24	(Rahayu et al., 2021)	Senior High School	Representation	Geometry	Active learning	Geometrical tools and	Test and non test	Indonesia	Q4

No.	Research (Year)	Level	Goal	Content	Process	Media	Assessment	Country	Journal
						representation materials			
25	(Marfuah et al., 2022)	Teacher	External transposition knowledge	Algebra	Structured online sessions	Geogebra	Test and non test	Indonesia	Q2
26	(Sumiaty & Dedy, 2019)	Undergraduate Student	Representation and communication	Algebra	Problem based learning	Worksheet	Test and non test	Indonesia	Q4
27	(Lutfi et al., 2021)	Junior High School	Learning obstacle	Geometry	Misrecognition of diagrammatic guidelines	Diagrams	Test and non test	Indonesia	Q4
28	(Ruli et al., 2019)	Junior High School	Learning obstacle	Algebra	Collaborative learning with contextual problem	Problem sheet	Test and non test	Indonesia	Q4
29	(Fuadiah & Suryadi, 2019)	Junior High School	Understanding	Number	Contextualized teaching	Number line and contextual scenarios	Non test	Indonesia	Q1
30	(Muin & Fatma, 2021)	Secondary School	Learning obstacle	Number, geometry, and algebra	Collaborative workshops and focus group discussions	Didactic material	Non test	Indonesia	Q4
31	(Hendriyanto et al., 2024)	Junior High School	Learning obstacle	Number	Didactic transposition	Didactic material	Test and non test	Indonesia	Q2
32	(Sukarma et al., 2024)	Junior High School	Learning obstacle	Number	A hybrid teaching approach	Learning material	Test and non test	Indonesia	Q1